Abstract—Speckle detection is essential in many areas of quantitative ultrasound. In this work, speckle is characterized with \( R = \text{SNR} \) and \( S = \text{skewness} \) of the amplitude of the ultrasound signal data \( A \). Different powers of \( A \) can be used to calculate \( R \) and \( S \). Prager et al. [1] proposed a method for finding the optimum power value, which then was further scrutinized [2]. We propose using two different powers of \( A \) in \( R \) and \( S \), and perform a large number of computer simulations to find these optimal values.

I. INTRODUCTION

Each pixel in an ultrasound image is formed by the back scattered echoes from an approximately ellipsoid called the resolution cell (Figure 1). The interference of scatterers in a resolution cell creates the granular appearance of the ultrasound image, called speckle. Although of random appearance, speckle pattern is identical if the same object is scanned from the same direction and under the same focusing and frequency.

Diffuse scattering happens if the scatterers in a resolution cell are placed independently and uniformly at random. If each resolution cell in an image patch has many such scatterers, the patch is said to be fully developed speckle (FDS). In contrast, white and dark features in the ultrasound B-mode images are caused by coherent back-scattering of ultrasound pulse.

Speckle detection is useful in segmentation, sensorless 3D freehand US, speckle cancellation and quantitative tissue characterization. In ultrasound compounding, for example, the goal is to cancel the speckles, while in sensorless 3D freehand ultrasound they are utilized to estimate probe movement.

Assume the effective number of scatterers per resolution cell to be \( \mu \), and the diffuse and coherent signal energy to be \( 2\sigma^2 \) and \( s^2 \) respectively. Speckles can be classified by \( \mu \) and \( k = \frac{s}{\sigma} \), with \( \mu > 10 \) and \( k < 1 \) being FDS.

Dutt et al. [3, 4] and Prager et al. [1] proposed using \( R \) and \( S \) to estimate \( \mu \) and \( k \) and therefore classify speckles

\[
R = \frac{\langle A^v \rangle}{\sqrt{\langle A^{2v} \rangle - \langle A^v \rangle^2}}
\]

\[
S = \frac{\langle (A^v - \langle A^v \rangle)^3 \rangle}{\langle A^{2v} \rangle - \langle A^v \rangle^2} \]  

where \( A \) is the amplitude of the ultrasound RF envelope, \( v \) is the signal power and \( \langle \cdots \rangle \) denotes mean. Depending on the correlation of data, thousands of sample data are required to reliably calculate \( R \) and \( S \) [4]. In [4] and [1], values of \( v \) that reduce this sample size are sought. Useful variability of clusters of sample data with different \( \mu \) and \( k \) values are maximized in [1] to find the optimal \( v \), a method that is scrutinized by [2].

Since \( R \) and \( S \) are different order moments of sample data, optimal values for \( v \) in \( R \) and \( S \) are not necessarily the same. We propose using different values of \( v \) in \( R \) and \( S \). We follow a similar approach to [1] to simulate the B-scan.

II. SIMULATION METHODS

We seek the optimal values of \( v_R \) and \( v_S \) that substitute \( v \) in equations 1 and 2 respectively. To create the sample data, the sum of \( \mu \) vectors of length \( \sqrt{2/\mu} \) and arbitrary phase (a random walk) is added to a single vector with zero phase and length \( k \), resulting in a vector \( A \), with amplitude \( A \).

The first step to compare the performance of different values for \( v_R \) and \( v_S \) is to obtain an FDS discriminant function. To this end, we set \( v_R = 0.2, 0.4 \ldots 3 \) and similarly \( v_S = 0.2, 0.4 \ldots 3 \), and for all combinations of \( v_R \) and \( v_S \) (15 \times 15 = 225 cases) acquire an FDS elliptical discrimination function in 3 steps (Figure 2):

1) 30000 sets of 5000 random \( A \) that represent FDS with different parameters \( 0 < k < 1 \) and \( 10 < \mu < 60 \) are
calculated (each data sample $A$ itself is obtained by the random walk described above, not shown in Figure 2).

2) For each set, $R$ and $S$ are calculated for all combinations of $0.2 \leq v_R \leq 3$ and $0.2 \leq v_S \leq 3$, resulting in $30000 \times 225$ samples of $R$ and $S$.

3) Using the 30000 samples of $R$ and $S$ for each $v_R$ and $v_S$ combination, 225 elliptical discriminant functions that encompass 95% of $R$ and $S$ values is obtained automatically using PCA and the covariance method.

Figure 3 shows the elliptical discriminant functions for $v_R = 0.2$ and $v_S = 0.2$ in the left images and for $v_R = 2$ and $v_S = 0.8$ in the right images. The two top images show 100 $(R, S)$ points that correspond to 100 sets of data $A$ with $\mu = 6$ and $k = 0$ values (few scatterers). These sets have to be categorized as non-FDS, therefore one can say that the pair $v_R = 2$ and $v_S = 0.8$ is performing better in this case (minimizing false acceptance). In the two bottom images, same $v_R$ and $v_S$ values are used, but for $\mu = 12$ and $k = 0$ (many scatterers, FDS). Both left and right discriminant functions categorize all 100 sets correctly (minimizing false rejection). This example shows that different $v_R$ and $v_S$ values affect the performance of the discriminant function.

In order to find the optimal $v_R$ and $v_S$ values, we obtain the probability that a set with properties $\mu$ and $k$ be identified as FDS, i.e. the $(R, S)$ pair calculated for this set falls inside the ellipse. Having such a pseudo pdf ($p_{FDS}$) for all values of $\mu$, $k$, $v_R$ and $v_S$, it is possible to optimize $v_R$ and $v_S$ to achieve a desired probability distribution. Obtaining $p_{FDS}$, which requires a large number of simulations, is as follows.

1) For all combinations of $k = 0, 0.1, 0.2 \cdots 1.5$ (16 values) and $\mu = 2, 3 \cdots 60$ (59 values) in the $k - \mu$ plane (Figure 4), generate 100 sets of 5000 random $A$. The result is $16 \times 59 \times 100 \times 5000$ random $A$, with each $A$ being calculated by a random walk.

2) Calculate $R$ and $S$ of each set for all combinations of $v_R$ and $v_S$.

3) $p_{FDS} = \text{number of the (R, S) pairs that fall inside the FDS discriminant ellipse. The resultant } p_{FDS} \text{ is a function of } v_R, v_S, \mu \text{ and } k$.

Different optimum values for $v_R$ and $v_S$ can be found depending on the criteria.

III. OPTIMIZING $v_R$ AND $v_S$

Using $p_{FDS}$ and depending on the particular application, different optimum values for $v_R$ and $v_S$ can be found. We analyze three cases here.

A. Minimizing False Acceptance for Coherent Sets

To minimize false acceptance for the coherent data, one can sum $p_{FDS}$ over the area $C$ in Figure 4. Figure 5 shows...
Fig. 4. The regions with $\mu < 10$ and $k > 1$ are labelled by $FS$ (few scatterers) and $C$ (coherent) respectively. The left shows the summation result. Generally speaking, the four corners of the $v_R - v_S$ plane should be avoided to prevent false acceptance. The FDS discriminant ellipse along with $(R, S)$ of 100 sets of 5000 sample $A$ for three values of $v_R = 0.4$, $v_S = 0.2$, $v_R = 3$, $v_S = 3$ (bad choices) and $v_R = 1.2$, $v_S = 0.6$ (good choice) are also shown.

Fig. 5. Top left: summation of $p_{FDS}$ over the area $C$ in Figure 4 as a function of $v_R$ and $v_S$. The bigger number indicates bigger false acceptance and therefore should be avoided. Top right and bottom left and right: The $R - S$ values of 100 sets of coherent data (each set has 5000 sample data $A$) for different $v_R$ and $v_S$ values as labelled. A more reliable discriminant function will classify less percentage of the points as FDS (bottom right).

B. Minimizing False Acceptance for Few Scatterer Sets

We sum $p_{FDS}$ over the area $FS$ in Figure 4 to minimize false acceptance of the sets with low scatterers (Figure 6 top left). The low values for $v_R$ and $v_S$ suffer from high false acceptance (Figure 6 top right and bottom left). A $v_R > 2$ and $v_S < 1.5$ value generates low false acceptance (Figure 6 bottom right).

C. Minimizing False Rejection for FDS Sets

To minimize the rejection of FDS sets, we sum $p_{FDS}$ over the area $FDS$ in Figure 4. Figure 7 shows the results.

IV. EXPERIMENTAL RESULTS

Based on the results of Figures 5 and 6, we conclude that in order to minimize false acceptance of non-FDS sets $v_S$ has to be approximately half of $v_R$. We performed the $R - S$ speckle detection method with different $v_R$ and $v_S$ values on bovine liver B-scans.

Figure 8 shows the image divided into patches of $100 \times 50$ pixels. The $R$ and $S$ values of each patch is calculated. The resultant point in the $R - S$ plane is connected to the center of the FDS discriminant ellipse and the ratio of the length of the line segment to the radius of ellipse at the intersection of the line segment and the ellipse is shown at the center of each patch. A patch with the ratio less than 1 can be considered as a FDS patch.

While the three different $v_R$ and $v_S$ values give similar results for the patches that are close to FDS, the pair $v_R = 2$ and $v_S = 1$ yields larger values for patches that are clearly not FDS (some samples marked by circle).
values as labelled. All three cases have to be approximately half of the sample power in $S$ which is specifically important for classification method discussed in this work, the sample power in $S$ has to be approximately half of the sample power in $R$. The values for $v_R$ and $v_S$ have to be selected according to specific concerns: minimizing false acceptance or false rejection or a combination of both. The three optimization criteria analyzed in this work provide a guideline for choosing appropriate values. Proper power selection can result in 50% more reliable classification.

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